Pressure Limiting Instabilities in Tokamaks

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Abstract

The plasma pressure achievable in a tokamak fusion reactor may be limited by instabilities like the ideal ballooning mode, a pressure-driven instability that acts to degrade plasma confinement. Here we investigate the sensitivity of the shape of the magnetic flux surface to the ideal ballooning mode; in particular, we modify the parameters describing the shape of magnetic flux surfaces of the equilibrium and perform infinite-n ideal ballooning scans to assess how shaping affects proximity to marginal instability. We find that for the parameter space considered, increasing squareness and elongation could help stabilise the plasma against the ideal ballooning mode instability.

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Introduction

Magnetic confinement fusion is a method used by tokamaks to maintain and control high-pressure plasma, with the ultimate goal of generating virtually limitless energy through nuclear fusion. In tokamaks, the magnetic field is helical around the torus due to the superposition of the poloidal (running around the plasma cross-section) and toroidal (running around the tokamak) magnetic fields. An important tokamak parameter is β, the ratio of plasma pressure p to magnetic field pressure, given by $\beta = 2\mu_0 p/B^2$, where μ_0 is the vacuum permeability and B is the magnetic field strength. While high β is desirable for efficiency, β is often limited by various instabilities in the plasma.

One type of pressure-driven electromagnetic instability is the ideal ballooning mode (Connor *et al.* 1978; Connor *et al.* 1979; Dewar *et al.* 1982). It is thought that this pressure-driven instability is important in high confinement (H-mode) plasmas, which is when the plasma is heated to reach a new regime of enhanced confinement. Specifically, this pressure-driven instability is of particular interest near the pedestal, a region of steep pressure gradient at the edge of the plasma that appears in H-mode (ASDEX Team 1989; Snyder *et al.* 2011; Dickinson *et al.* 2012). This study aimed to explore the impact of shaping of tokamak magnetic equilibria on ideal ballooning stability in H-mode plasmas, using data from the MAST and MAST Upgrade (MAST-U) spherical tokamaks at the UK Atomic Energy Authority (UKAEA).

Theory

This analysis utilises ideal magnetohydrodynamics (ideal MHD), a description of the plasma as a single fluid assuming quasineutrality and neglecting resistivity (Freidberg 2014). For axisymmetric equilibria, the magnetic field lines lie on nested toroidal magnetic flux surfaces of constant pressure (Wesson 2011). In equilibrium:

$$\boldsymbol{j} \times \boldsymbol{B} = \nabla p \ , \tag{1}$$

where $\boldsymbol{j}$ is the current density and $\boldsymbol{B}$ is the magnetic field. Equation 1 shows that there is no pressure gradient ∇p along the magnetic field lines. The equilibrium for an axisymmetric ideal MHD system can alternatively be written as the Grad-Shafranov differential equation (Wesson 2011). The solution to this equation gives the toroidal current density and poloidal magnetic field across the plasma equilibrium, allowing the calculation of the local equilibria on any flux surface in the plasma.

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The ideal ballooning equation can be used to determine if the plasma on a specific flux surface is stable to the ideal ballooning mode. We describe the perturbation from equilibrium with the local displacement vector $\boldsymbol{\xi}(\boldsymbol{r}, t)$, which represents the displacement of the plasma from its equilibrium position as a function of space ($\boldsymbol{r}$) and time (t). The derivation of the ideal ballooning equation (Roach 2023; Cowley 2024) decomposes the perturbations as Fourier modes to be of the form $\boldsymbol{\xi} = e^{-i\omega t}e^{inS}\hat{\boldsymbol{\xi}}$ where ω is frequency, n is the mode number and S is the field line label in a field-aligned coordinate system (Dudding 2022). In this analysis, we take the limit of $n \to \infty$ and assume incompressible perturbations. The ideal ballooning equation is an equation of motion which, when solved, gives us ω^2. If we find $\omega^2 < 0$ then ω must be imaginary so that an unstable eigenmode solution (i.e. an instability) exists that grows exponentially in time by extracting energy from the pressure gradient and magnetic field curvature, thereby limiting plasma confinement.

Methods

Local Equilibrium

The balance between the local forces on the plasma (Equation 1) maintains plasma equilibrium, which can be described as a set of magnetic flux surfaces. Equilibrium reconstruction for a tokamak experiment is carried out by solving the Grad-Shafranov equation on a 2D rectangular grid at a constant toroidal angle in the poloidal plane, essentially taking a cross-section of the plasma in the torus. The equation is solved numerically using Equilibrium reconstruction and fitting code (EFIT) (Lao *et al.* 1985) and experimental measurements (Wesson 2011) as constraints to the solution. For the small scale, highly localised instabilities studied here ($n = \infty$), only the details of the plasma from an extremely narrow layer around a given flux surface are needed.

We take the full 2D equilibrium reconstructions from EFIT and use pyrokinetics code (Patel *et al.* 2024) to extract information from the specified flux surfaces. This approach utilises an analytic parameterisation (Miller *et al.* 1998; Dudding 2022) of the flux surface shape, giving us the Miller flux surface shaping parameters. The parameters we will focus on for this study include the elongation κ_M, triangularity δ_M, and squareness ζ_M (Turnbull *et al.* 1999). The radial derivatives of these parameters are fit using the Grad-Shafranov solution to derive flux surface plots (Figure 1a, c, and e).

Ballooning Solver

Pyrokinetics (Pyro; Patel *et al.* 2024) is a Python package written to standardise calculations of small-scale instabilities in tokamak plasmas. The infinite-n ideal ballooning solver in pyrokinetics was developed by Rahul Gaur (Gaur *et al.* 2023) using an adjoint-based method that solves the derivatives of the ballooning equation with respect to all inputs of the system (Giannakoglou *et al.* 2008). This project uses Pyro to extract the relevant information from EFIT, compute the Miller parameters for local flux surfaces (using an optimisation algorithm), calculate the full local equilibrium, and finally use all of this data in the ballooning solver to solve for ω^2.

It is important to note that the ballooning equation solves for stability only within a localised volume near a specified flux surface, and that the stability of the ideal ballooning mode varies with flux surface, which we label with the normalised poloidal flux ψ_N. To analyse the stability of the ideal ballooning mode across the whole plasma cross-section we applied the solver over a range of flux surfaces, with data for each surface taken from a global Grad-Shafranov equilibrium solution.

The experimental tokamak plasmas we are examining are not expected to be maintained in a state unstable to ideal ballooning because, if unstable, the ideal ballooning mode causes transport losses of matter and energy that reduces the local pressure gradient to a point where the mode becomes stable again. It is believed that these instabilities limit the size of the pressure gradient possible in certain circumstances, for example in the pedestal of H-mode plasmas. It is particularly interesting to assess the proximity of local equilibria to the stability boundary (where $\omega^2 = 0$) of the ideal ballooning mode. We can do this by computing ω^2 for a range of a parameter that defines the local equilibrium. The ideal ballooning mode is driven by the pressure gradient so it is typical to vary the normalised pressure gradient, β' (defined as $\beta' = \frac{d\beta}{dr}$ where r is the radial coordinate) to assess proximity to marginal stability.

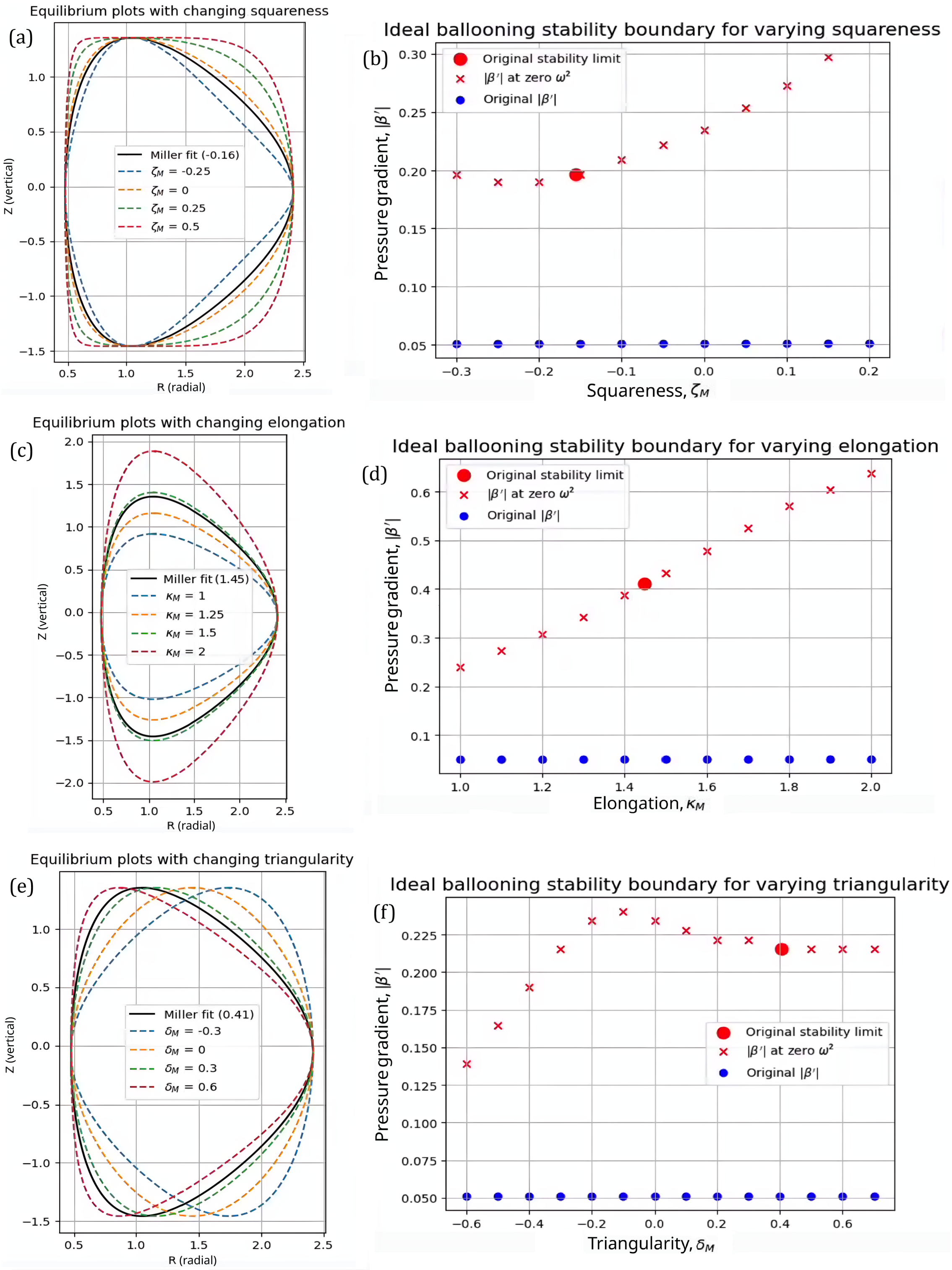

Figure 1: (**a**), (**c**), (**e**): flux surface $\psi_N = 0.97$ for MAST shot 30422 at 326ms, where the solid black line is the Miller parameter fit to the experimental data and the dashed lines are the modified shaping parameters where all other geometry parameters are held to their original values. (**b**), (**d**), (**f**): β' against shaping parameter ζ_M (**b**), κ_M (**d**), and δ_M (**e**) with experimental values as blue dots, the boundary of stability as red crosses and the original shaping and corresponding β' parameter for the flux surface as a red dot. Plot (**b**) at $\zeta_M = 0.2$ does not have a corresponding critical β' value as there is no limit with these particular experimental parameters when only increasing β'.

Choice of Experimental Run

The experimental run we choose to analyse in this study is MAST's 30422. This experimental discharge (shot) is used in a previous study of pedestal stability regimes (Imada *et al.* 2024), which analyses the differences in stability to the ballooning mode between MAST and MAST-U plasmas and mainly discusses finite-n MHD ballooning modes. Here we expand on that work by exploring $n = \infty$ ballooning modes in more detail.

We analyse the 326 ms time slice because time trace analysis for this shot suggests that the plasma is most stable at that time. Pressure profile analysis indicates that the plasma is operating in H-mode due to the presence of the pedestal at roughly $\psi_N = 0.95$. The surface at 0.97 has the highest experimental β' and by making β' scans across the plasma cross-section, we can see that it is one of the closest flux surfaces to the limit of stability, making it a good candidate for analysis.

Results

We next explore how shaping can influence the sensitivity of ideal ballooning modes by varying the shaping parameters δ_M, κ_M, ζ_M of flux surface $\psi_N = 0.97$. Figure 1 suggests that increasing ζ_M and κ_M shifts the stability boundary to higher β' values and therefore acts to make $n = \infty$ ideal ballooning modes more stable. This aligns with a similar analysis we carried out on the MAST-U discharge 45272, which itself has higher κ_M and ζ_M values. Here, a higher pedestal and an increased stability against the ideal ballooning mode was observed when compared to the MAST discharge.

Discussion

A limitation of this study is that such shaping scans are difficult to replicate in experimental plasmas and, generally, only the plasma boundary shape can be directly modified by tweaking the toroidal and poloidal field coils. Furthermore, the internal local flux surfaces must self-consistently satisfy the Grad-Shafranov equation for a given boundary shape, which we do not take into account as we only focus on one surface. Future directions for this work include exploring ideal ballooning mode sensitivity to shaping of the last closed flux surface using fully self-consistent Grad-Shafranov solutions to analyse the full range of flux surfaces for a given time slice (which would be more comparable to experimental data).

Conclusion

This project explored the potential of using the ideal ballooning analysis to inform designers of future tokamaks which shapes of magnetic flux surfaces may be more favourable than others. Local ballooning analysis supports the notion that shaping, especially squareness and elongation, is related to the higher performing pedestals recorded in MAST-U compared to those of MAST. Specifically, it seems to be the increase of squareness and elongation that moves the marginal stability boundary away from the experimental equilibria, thus potentially leading to improved plasma confinement. There was insufficient time to rigorously test whether this method or the extracted conclusions are consistent with observed tokamak plasmas as the shaping study used toy models of the plasma which have not been compared to real discharges. Future work should consider an extended parameter space to understand the generality of these observations, as well as analyse globally consistent solutions of the Grad-Shafranov equation.

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